given by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=v^{v/(v-1)} \text{ and } \frac{z^2}{c^2}=v^{1/(v-1)}-1.$$

This surface is symmetrical with reference to the plane $z=0$, and incloses that plane by an infinite elliptic boundary. As v increases from 0 to ∞ the sections parallel to $z=0$ decrease continually, remaining ellipses always, until they reach their limiting size, which is that of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Another companion to the same curve is

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=v^{v/(v-1)} \text{ and } \frac{y^2}{b^2}=1-v^{1/(v-1)}.$$

y has a real value only when $v^{1/(v-1)}=1$. Hence the companion degenerates to $y=0$ and $\frac{x^2}{a^2}-\frac{z^2}{c^2}=\infty$. It is therefore an hyperbola infinitely removed from the origin and lying in the plane $y=0$.

The companion to the sphere $x^2+y^2+z^2=a^2$ is

$$x^2+y^2=v^{v/(v-1)} \text{ and } z^2=a^2-v^{1/(v-1)}.$$

When $a^2=1$, the companion is an infinite circle. When $a^2<1$, the companion is imaginary. When $a^2>1$, the companion is a real surface. Every section by planes parallel to $z=0$ is a circle. For all values of v which make $v^{1/(v-1)}>a^2$ the planes which make circular sections are imaginary.

NOTE ON THE ADDITION THEOREM IN TRIGONOMETRY.

By DR. G. A. MILLER.

When $\cos\alpha$, $\cos\beta$, $\sin\alpha$, $\sin\beta$ are substituted for x_1, x_2, y_1, y_2 respectively, in the well known identity

$$(x_1^2+y_1^2)(x_2^2+y_2^2)=(x_1y_2+x_2y_1)^2+(x_1x_2-y_1y_2)^2,$$

there results

$$(\cos^2\alpha+\sin^2\alpha)(\cos^2\beta+\sin^2\beta)=(\cos\alpha\sin\beta+\cos\beta\sin\alpha)^2+(\cos\alpha\cos\beta-\sin\alpha\sin\beta)^2$$

or $$(\sin\alpha\cos\beta+\cos\alpha\sin\beta)^2+(\cos\alpha\cos\beta-\sin\alpha\sin\beta)^2=1.$$

For every value of and β the two expressions in parenthesis represent real

numbers whose squares are together equal to unity. Hence one represents $\sin\phi$ and the other $\cos\phi$, where ϕ is a function of α and β.

Suppose we know that

$$\sin\alpha\cos\beta+\cos\alpha\sin\beta=\sin(\alpha+\beta)=\sin(180^\circ-\alpha-\beta) \qquad (A).$$

It follows from the given identity that

$$\cos\alpha\cos\beta-\sin\alpha\sin\beta=\cos(\alpha+\beta) \text{ or } \cos(180^\circ-\alpha-\beta).$$

The latter is impossible since $\alpha=\beta=0$ would not satisfy the equation. Hence we have that from the given identity and (A) it follows that

$$\cos\alpha\cos\beta-\sin\alpha\sin\beta=\cos(\alpha+\beta)=\cos(-\alpha-\beta).$$

If the given identity is written in the form

$$(x_1^2+y_1^2)(x_2^2+y_2^2)=(x_2y_1-x_1y_2)^2+(x_1x_2+y_1y_2)^2$$

we have, by the method used above, that

$$\sin\alpha\cos\beta-\cos\alpha\sin\beta \text{ and } \cos\alpha\cos\beta+\sin\alpha\sin\beta$$

are respectively the sign and cosine of a given angle. It should be observed that it does not matter which of these expressions is taken for sine ϕ except that the formulas come out in a little briefer form if we proceed as above.

This note does not aim at completeness. Its object is to call the attention of teachers of trigonometry to the fact that the derivation of these formulas could proceed along lines which differ from those which are generally pursued, and that these methods offer many advantages. Other things being equal it is always desirable to exhibit connection between different formulas rather than to derive each independently.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

229. Proposed by BENJAMIN FRANKLIN YANNEY, Mount Union College, Alliance, Ohio.

If $a_1^n+a_2^n+a_3^n+\ldots+a_r^n=A^n$, $a_1^m+a_2^m+a_3^m+\ldots+a_r^m>$ or $<A^m$, according as $m<$ or $>n$; provided all the letters stand for positive real numbers.